OCR
Oxford Cambridge and RSA

Friday 24 June 2016 – Morning

A2 GCE MATHEMATICS

4724/01 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4724/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 Find the quotient and the remainder when $4x^3+8x^2-5x+12$ is divided by $2x^2+1$. **[3]**

2 Use integration to find the exact value of $\int_{\frac{1}{16}\pi}^{\frac{1}{8}\pi}(9-6\cos^2 4x)\,dx$. **[5]**

3 Given that $y\sin 2x+\frac{1}{x}+y^2=5$, find an expression for $\frac{dy}{dx}$ in terms of x and y. **[5]**

4 Find the exact value of $\int_1^8 \frac{1}{\sqrt[3]{x}}\ln x\,dx$, giving your answer in the form $A\ln 2+B$, where A and B are constants to be found. **[5]**

5 The vector equations of two lines are as follows.

$$L\colon \mathbf{r}=\begin{pmatrix}1\\4\\5\end{pmatrix}+s\begin{pmatrix}2\\-1\\3\end{pmatrix} \qquad M\colon \mathbf{r}=\begin{pmatrix}3\\2\\-5\end{pmatrix}+t\begin{pmatrix}5\\-3\\1\end{pmatrix}$$

(i) Show that the lines L and M meet, and find the coordinates of the point of intersection. **[4]**

(ii) Show that the line L can also be represented by the equation $\mathbf{r}=\begin{pmatrix}7\\1\\14\end{pmatrix}+u\begin{pmatrix}-4\\2\\-6\end{pmatrix}$. **[2]**

6 Use the substitution $u=x^2-2$ to find $\int \frac{6x^3+4x}{\sqrt{x^2-2}}\,dx$. **[6]**

7 Given that the binomial expansion of $(1+kx)^n$ is $1-6x+30x^2+\ldots$, find the values of n and k. State the set of values of x for which this expansion is valid. **[6]**

8 The points A and B have position vectors relative to the origin O given by

$$\overrightarrow{OA} = \begin{pmatrix} 3\sin\alpha \\ 2\cos\alpha \\ -1 \end{pmatrix} \text{ and } \overrightarrow{OB} = \begin{pmatrix} 2\cos\alpha \\ 4\sin\alpha \\ 3 \end{pmatrix},$$

where $0° < \alpha < 90°$. It is given that $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are perpendicular.

(i) Calculate the two possible values of α. **[5]**

(ii) Calculate the area of triangle OAB for the smaller value of α from part **(i)**. **[4]**

9 A curve has parametric equations $x = 1 - \cos t$, $y = \sin t \sin 2t$, for $0 \leqslant t \leqslant \pi$.

(i) Find the coordinates of the points where the curve meets the x-axis. **[3]**

(ii) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2\cos 2t + 2\cos^2 t$. Hence find, in an exact form, the coordinates of the stationary points. **[7]**

(iii) Find the cartesian equation of the curve. Give your answer in the form $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is a polynomial. **[3]**

(iv) Sketch the curve. **[2]**

10 **(i)** Express $\frac{16 + 5x - 2x^2}{(x+1)^2(x+4)}$ in partial fractions. **[5]**

(ii) It is given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(16 + 5x - 2x^2)y}{(x+1)^2(x+4)}$$

and that $y = \frac{1}{256}$ when $x = 0$. Find the exact value of y when $x = 2$. Give your answer in the form $A\mathrm{e}^n$. **[7]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			$2x$ seen in quotient and $4x^3 + 2x$ seen in division	**B1**		if **B0M0, B2** for quotient is $2x + 4$ or for remainder is $-7x + 8$; **B3** for both of these
			$8x^2 + kx$ [+ 12] seen in division	**M1**	**NB** $k = -7$	
			$2x + 4$ seen and $-7x + 8$ seen isw	**A1**		ignore wrong labelling
				[3]		
2			$\cos 8x$ seen in integrand	**M1**		
			$\mathrm{F}[x] = Ax + B\sin 8x$ oe	**M1***	A and B are non-zero constants	
			$\mathrm{F}[x] = 6x - \frac{3}{8}\sin 8x$	**A1**		
			$\mathrm{F}[\frac{1}{8}\pi] - F[\frac{1}{16}\pi]$	**M1*dep**		
			$\frac{3}{8}\pi + \frac{3}{8}$ oe	**A1**		allow eg $0.375\pi + 0.375$ or fractions not in lowest terms
				[5]		

Question			Answer	Marks	Guidance	
3			$2y\frac{\mathrm{d}y}{\mathrm{d}x}$	B1	from differentiation of y^2	
			$\sin 2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y\cos 2x$	M1	correct use of Product Rule	allow sign error or one incorrect coefficient
			$\sin 2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y\cos 2x-\frac{1}{x^2}+2y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	A1		
			$(\sin 2x+2y)\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x^2}-2y\cos 2x$ oe	M1	collection of like terms on separate sides, need not be factorised	must be two terms in $\frac{\mathrm{d}y}{\mathrm{d}x}$
			$[\frac{\mathrm{d}y}{\mathrm{d}x}=]\frac{1-2x^2y\cos 2x}{(\sin 2x+2y)x^2}$ oe isw	A1 [5]	eg $[\frac{\mathrm{d}y}{\mathrm{d}x}=]\frac{x^{-2}-2y\cos 2x}{(\sin 2x+2y)}$	**A0** for eg $y=\ldots$
4			$Ax^{\frac{2}{3}}\ln x-\int Bx^{\frac{2}{3}}\times\frac{1}{x}\mathrm{d}x$ oe	M1*	A and B are non-zero constants;	
			$\frac{3}{2}x^{\frac{2}{3}}\ln x-\int\frac{3}{2}x^{\frac{2}{3}}\times\frac{1}{x}\mathrm{d}x$	A1	ignore $+\ c$	NB $\frac{3}{2}x^{\frac{2}{3}}\ln x-\int\frac{3}{2}x^{-\frac{1}{3}}\mathrm{d}x$ Allow both marks if dx omitted
			$\mathrm{F}[x]=\frac{3}{2}x^{\frac{2}{3}}\ln x-\frac{3/2}{2/3}x^{\frac{2}{3}}$	A1	ignore limits for first three marks	
			F[8] – F[1]	M1*dep	and also dependent on integration of their $\frac{3}{2}x^{-\frac{1}{3}}$	
			$18\ln 2-\frac{27}{4}$ cao	A1 [5]		**NB A0** for $6\ln 8-\frac{27}{4}$

Question		Answer	Marks	Guidance	
5	(i)	$3 + 5t = 1 + 2s$ $2 - 3t = 4 - s$ $-5 + t = 5 + 3s$	M1	attempt to solve any two of these simultaneously to obtain a value of s or t	
		$t = -2$ and $s = -4$	A1		
		substitution of their s and t in other equation to obtain eg $-7 = -7$ oe (1st or 3rd equation) or eg $8 = 8$ oe (2nd equation)	B1	may be embedded, eg $-5 + -2 = 5 + 3 \times -4$	**B0** if any subsequent arithmetic errors seen eg $-5 + -2 = 5 + 3 \times -4$ so $7 = -7$
		lines meet at $(-7, 8, -7)$	A1	allow in vector form	
			[4]		
5	(ii)	$\begin{pmatrix} -4 \\ 2 \\ -6 \end{pmatrix} = -2\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ oe seen	B1	do not allow eg $\begin{pmatrix} -4 \\ 2 \\ -6 \end{pmatrix} \div \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = -2$	or $\begin{pmatrix} 7 \\ 1 \\ 14 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 9 \end{pmatrix}$ $= 3\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$
		common point identified and justified eg by substitution of correct value of s or u eg $s = 3$ or eg $u = \frac{3}{2}$	B1		
			[2]		
		Alternatively		or eg $u = \frac{3}{2}s - \frac{1}{2}$	or
		substitution of eg $s = 3 - 2u$	B1		show one pair of equations consistent
		and completion to $\mathbf{r} = \begin{pmatrix} 7 \\ 1 \\ 14 \end{pmatrix} + u\begin{pmatrix} -4 \\ 2 \\ -6 \end{pmatrix}$	B1	and completion to $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} + u\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$	show another pair consistent
			[2]		

Question			Answer	Marks	Guidance	
6			$\frac{du}{dx} = 2x$ oe or $\frac{dx}{du} = \frac{1}{2}(u \pm 2)^{-\frac{1}{2}}$ oe	M1		
			$\frac{Ax^2 + B}{2}$ or better from replacing dx NB $\frac{6x^3 + 4x}{2x} = \frac{6x^2 + 4}{2}$	M1		or substitution of $x = (u \pm 2)^{\frac{1}{2}}$ in denominator from $\frac{dx}{du}$
			substitution of $x^2 = u \pm 2$ or $x = (u \pm 2)^{\frac{1}{2}}$ in numerator	M1	**NB** $3(u+2)+2$ or $3(u+2)^{\frac{3}{2}} + 2(u+2)^{\frac{1}{2}}$	
			$\int (\frac{3u+8}{\sqrt{u}})[du]$ oe	A1	$\frac{3(u+2)+2}{\sqrt{u}}$ or better	
			$\frac{3u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{8u^{\frac{1}{2}}}{\frac{1}{2}}$ oe	A1	or $6u^{\frac{3}{2}} + 16u^{\frac{1}{2}} - 4u^{\frac{3}{2}}$ from integration by parts	
			$2(x^2-2)^{\frac{3}{2}} + 16(x^2-2)^{\frac{1}{2}} + c$ cao	A1 **[6]**	allow $2(x^2-2)^{\frac{1}{2}}(x^2+6)+c$ for final mark, **A0** if du not seen at some stage in the integral	must see constant of integration here or in previous line and coefficients must be simplified for final **A1**

Question			Answer	Marks	Guidance	
7			$nk = -6$ soi	B1	allow $nkx = -6x$ and /or $\frac{n(n-1)k^2}{2!}x^2 = 30x^2$ for first two marks	
			$\frac{n(n-1)k^2}{2!} = 30$ soi	B1		**NB** $\frac{n(n-1)\times 36}{2\times n^2} = 30$ oe $(-\frac{6}{k})(\frac{-6}{k}-1)k^2 = 60$ oe
			substitution of $n = \pm\frac{6}{k}$ or $k = \pm\frac{6}{n}$ or k= $\pm\sqrt{\frac{60}{n(n-1)}}$ oe to eliminate one variable from their equations	M1	allow omission of brackets	
			$n = -1.5$ oe	A1	eg allow $-\frac{6}{4}$	
			$k = 4$	A1		
			expansion is valid for $\lvert x\rvert < \frac{1}{4}$ or $-\frac{1}{4} < x < \frac{1}{4}$ isw	B1FT **[6]**	**FT** their k	
8	(i)		$3\sin\alpha \times 2\cos\alpha + 2\cos\alpha \times 4\sin\alpha + -1\times 3$	M1		allow one sign error or one coefficient error for **M1**
			$6\sin\alpha\cos\alpha + 8\sin\alpha\cos\alpha - 3 = 0$ soi	A1		
			substitution of sinα cosα = ½ sin 2α oe	M1	**NB** 7sin 2α = 3	or squaring both sides and correct substitution from Pythagoras
			α = awrt 12.7°	A1	awrt 0.221	
			α = awrt 77.3°	A1 **[5]**	awrt 1.35	if **A0A0**, **SC1** for 13° and 77° or 0.22 and 1.4

Question		Answer	Marks	Guidance	
8	(ii)	their α = 12.7° substituted in $\overrightarrow{OA}$ and $\overrightarrow{OB}$; or in $\lvert\overrightarrow{OA}\rvert$ and $\lvert\overrightarrow{OB}\rvert$	M1		**NB** $\begin{pmatrix} 0.6589.. \\ 1.9511.. \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1.9511.. \\ 0.8785.. \\ 3 \end{pmatrix}$
		$\sqrt{(3\sin\alpha)^2+(2\cos\alpha)^2+(-1)^2}$ or $\sqrt{(2\cos\alpha)^2+(4\sin\alpha)^2+3^2}$	M1*	allow omission of brackets, one slip in arithmetic and one sign error;	
		$\frac{1}{2}\sqrt{9\sin^2\alpha+4\cos^2\alpha+1}\sqrt{4\cos^2\alpha+16\sin^2\alpha+9}$	M1*dep	may be implied by numerical value for lengths; allow one sign or coefficient error *α* may be unspecified or any acute angle for these method marks	**NB** √5.241..= 2.289… and √13.579.. = 3.685…
		awrt 4.22	A1 [4]		**NB** hypotenuse is 4.34 and other angles in triangle are 58.2° and 31.8°
9	(i)	sin*t*sin2*t* = 0 oe seen	M1		**NB** *t* = 0, ½π, π
		(0, 0) (1, 0) and (2, 0) or *x* = 0, *x* = 1, *x* = 2 cao	A2 [3]	**A1** for two of three correct	deduct 1 mark if all three correct plus extra values if **A0**, allow **SC1** for *t* = 0, ½π, π if unsupported, full marks for all three values correct

Question		Answer	Marks	Guidance	
9	(ii)	$\left[\frac{\mathrm{d}y}{\mathrm{d}t}\right] = 2\sin t\cos 2t + \cos t\sin 2t$	**B1**	or $4\sin t\cos^2 t - 2\sin^3 t$	
		$\frac{(2\sin t\cos 2t + \cos t\sin 2t)}{\sin t}$ or $\frac{(4\sin t\cos^2 t - 2\sin^3 t)}{\sin t}$	**M1**	allow sign errors and/or one incorrect coefficient	
		substitution of $\sin 2t = 2\sin t\cos t$ in their	**M1**	may be seen before differentiation	
		$\frac{(2\sin t\cos 2t + \cos t\sin 2t)}{\sin t}$ and completion to $2\cos 2t + 2\cos^2 t$ www **NB AG**	**A1**	at least one correct intermediate step needed	
		eg $2(2\cos^2 t - 1) + 2\cos^2 t = 0$ or $2\cos 2t + 2\times\frac{1}{2}(1 + \cos 2t) = 0$	**M1**	use of double angle formula to obtain quadratic equation in eg $\cos t$ or linear equation in $\cos 2t$; may be seen before differentiation	mark intent: allow sign error, bracket error, omission of one coefficient
		$(1+\frac{1}{\sqrt{3}}, \frac{-4}{3\sqrt{3}})$ oe isw	**A1**		eg $(\frac{\sqrt{3}+3}{3}, -\frac{4\sqrt{3}}{9})$
		$(1-\frac{1}{\sqrt{3}}, \frac{4}{3\sqrt{3}})$ oe isw	**A1** [7]	if **A0**, **A0**, allow **A1** for both x values correct	

Question		Answer	Marks	Guidance	
9	(iii)	$y = 2(1 - \cos^2 t)\cos t$ oe may be implicit equation, may be implied by partial substitution for $\cos t$ eg $(1-x)^2 + \dfrac{y}{2\cos t} = 1$	M1	or $\dfrac{dy}{dx} = 6\cos^2 t - 2$	use of double angle formula (and Pythagoras) to obtain expression for y or $\dfrac{dy}{dx}$ in terms of $\cos t$ only;
		$y = 2(1 - (1-x)^2)(1 - x)$	M1	or $\dfrac{dy}{dx} = 6(1-x)^2 - 2$	substitution of $\cos t = \pm 1 \pm x$ to obtain expression in terms of x only allow sign errors, bracket errors or minor slips in arithmetic eg omission of 2 for these method marks
		$y = 2x^3 - 6x^2 + 4x$ or $y = 2x(x^2 - 3x + 2)$ or $y = 2x(x-1)(x-2)$ oe cao	A1 **[3]**	integration and substitution of eg (0, 0) to obtain correct answer must see $y =$ at some stage for **A1**	
9	(iv)	cubic with two turning points and of correct orientation through (0, 0) x-intercepts correct and only for $0 \le x \le 2$	M1 A1 **[2]**		

Question			Answer	Marks	Guidance	
10	(i)		$\dfrac{A}{(x+4)}+\dfrac{B}{(x+1)}+\dfrac{C}{(x+1)^2}$	B1		may be awarded later
			$[16+5x-2x^2]=A(x+1)^2+B(x+1)(x+4)+C(x+4)$	M1		allow sign errors only
			$A=-4$	A1	**NB** $36=-9A$	if **B0M0,** allow **SC3** for $\dfrac{2x+5}{(x+1)^2}-\dfrac{4}{x+4}$
			$C=3$	A1	$9=3C$	
			$B=2$ isw	A1	$-2=A+B$, $5=2A+5B+C$ $16=A+4B+4C$	
				[5]	**NB** $\dfrac{-4}{(x+4)}+\dfrac{2}{(x+1)}+\dfrac{3}{(x+1)^2}$	

Question		Answer	Marks	Guidance	
10	(ii)	$\int \frac{\mathrm{d}y}{y} = \int \frac{16+5x-2x^2}{(x+1)^2(x+4)}\,\mathrm{d}x$	**B1**	separation of variables	allow omission of integral signs; allow omission of dy or dx but not both
		$\frac{3}{(x+1)^2} + \frac{2}{(x+1)} - \frac{4}{(x+4)}$ seen in RHS, may be embedded	**M1***	**FT** their partial fractions if two or three terms; ignore LHS	may be implied by correct integration of two of their terms
		$\frac{-3}{x+1} + 2\ln(x+1) - 4\ln(x+4) + c$	**A1FT**	**FT** their non-zero 3, 2 and 4; allow recovery from $x + 1^2$ in denominator; if brackets in log terms omitted, allow **A1** if recovery seen in substitution	allow omission of $+ c$ here
		$\ln(\frac{1}{256}) = -3 + 2\ln 1 - 4\ln 4 + c$	**M1*dep**	substitution of $x = 0$ and $y = \frac{1}{256}$; allow if error in manipulation following integration;	$+ c$ must be included and LHS must be correctly obtained
		$c = 3$ cao	**A1**	or $A = \mathrm{e}^{-3}$ from $y = Ae^{\frac{-3}{x+1}}\frac{(x+1)^2}{(x+4)^4}$	
		$\ln y = \frac{-3}{2+1} + 2\ln(2+1) - 4\ln(2+4) + 3$	**M1*dep**	substitution of $x = 2$; dependent on award of previous **M1M1** and numerical value found for c	allow **M1** if substitution follows incorrect manipulation eg to find expression for y
		$y = \frac{e^2}{144}$ oe	**A1** [7]		